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Efect of Wall Thermal Properties on the Mean Temperature Profile in Near-Wall Turbulence

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Abstract - Heat transfer coefficient in the fully developed turbulent channel flow bounded with walls heated with constant heat flux, is known to be a function of Reynolds and Prandtl number only (if the temperature is assumed to be a passive scalar). The present study, based on the results of the Direct Numerical Simulations reports a small difference of approximatelly 0.5% that exists in heat transfer rates near the wall of fully developed turbulent channel flow at the same Reynolds and Prandtl numbers, the same heat flux, but with different thermal activity ratios $K = \infty$ and $K = 0$ ($K=\sqrt{(\rho_f c_{pf} \lambda_f)/(\rho_w c_{pw} \lambda_w)}$).

1. Introduction

Direct Numerical Simulation (DNS) is an important tool for the investigation of near-wall turbulent heat transfer. Results in this field were obtained by Kasagi et al. [1], Kim and Moin [2], Kawamura et al. [3], Kasagi and Iida [4] and Na et al. [5]. Calculations presented in these papers were carried out in a channel flow with uniform heating of the walls, except in the work of Kim and Moin [2], where the volumetric heating of the fluid was assumed. Similar DNS study of passive scalar transfer in the flume geometry, where the flow is limited with a single heated wall and a free surface, was performed by Tiselj et. al. [6].

2. Equations and Boundary Conditions

As the problem was well studied in the past, the governing equations and the scaling procedure can be found elsewhere [1,2,3,4,5,6]. The same equations are valid in the channel and flume geometry. The dimensionless form (in wall units) of the energy equation is given as follows:

$$\frac{\partial \theta^+}{\partial t^+} = -\nabla^+ \cdot \left(\vec{u}^+ \theta^+ \right) + \frac{1}{Pr} \nabla^{+2} \theta^+ + \frac{1}{Re_\tau} \frac{u_x^+}{u_B^+}, \tag{1}$$

where are x, y, and z are streamwise, wall-normal, and spanwise directions, respectively. The dimensionless temperature θ^+ (correct expression would be actually "dimensionless temperature difference") is defined as:

$$\theta^+(x,y,z,t) = \left\{ \langle T_{WALL} \rangle_{t,z}(x) - T(x,y,z,t) \right\} / T_\tau \tag{2}$$

The averaging sign $<.>$ with subscripts t and z means ensemble averaging over the spanwise direction and time. Wall temperature $< T_{WALL} >_{t,z}$ is increasing linearly in the streamwise direction, while the profile of the dimensionless temperature $<\theta^+>_{t,z}$ remains constant in streamwise direction; ($<\theta^+>_{t,z} = <\theta^+>_{t,z,x}$). The friction temperature is defined as $T_\tau = q_w / \rho_f c_{pf} u_\tau$ and the friction Reynolds number as $Re_\tau = u_\tau \delta / \nu$, where q_w is the wall heat flux, u_τ the friction velocity and δ is the channel half-width or the flume height (δ in wall units is $\delta^+ = Re_\tau$). Term u_x^+ / u_B^+ appears in Eq. (1) due to the transformation of the temperature T into the periodic variable θ^+ . Bulk streamwise velocity is defined as $u_B^+ = <u_x^+>_{t,x,y,z}$.

Periodic boundary conditions are applied in streamwise and spanwise directions. Free surface in flume geometry was assumed to be free slip and adiabatic. Definitions of the wall boundary conditions for the dimensionless temperature θ^+ are the following:

- The isothermal boundary condition for the temperature θ^+ sets the wall temperature to zero: $\theta^+(x, y = y_{WALL}, z, t) = 0$. If the dimensionless temperature θ^+ is split into the mean $<\theta^+>_{t,x,z}(y)$ and the fluctuating part $\theta'^+(x, y, z, t)$, the same boundary condition can be applied to both parts: $<\theta^+>_{t,x,z}(y = y_{WALL}) = 0$ and $\theta'^+(x, y = y_{WALL}, z, t) = 0$. This type of boundary condition was used in the previous DNS studies [1,2,3,4,5].
- The isoflux boundary condition for the temperature θ^+ sets the mean dimensionless temperature at the wall to zero: $<\theta^+>_{t,x,z}(y = y_{WALL}) = 0$, but the boundary condition for the fluctuating part θ'^+ at the wall-fluid interface is different: $d\theta'^+/dy\ (x, y = y_{WALL}, z, t) = 0$. Implementation of the isoflux boundary condition for θ^+ in pseudo-spectral scheme is discussed by Tiselj et. al. [6].

The boundary condition on the real heated wall should be actually obtained with the conjugate heat transfer calculation, which takes into account heat conduction inside the wall. As shown by Kasagi et al. [7] and Tiselj et al. [8], who performed such conjugate heat transfer calculations in the channel, type of the thermal boundary condition in an experimental setup depends on the thermal activity ratio $K = \sqrt{(\rho_f c_{pf} \lambda_f)/(\rho_w c_{pw} \lambda_w)}$ that combines material properties of fluid (subscript *f*) and heater (subscript *w*). Isothermal and isoflux boundary conditions for dimensionless temperature θ^+ represent two limiting cases with $K = 0$ and $K = \infty$, respectively. In experiments with air the boundary condition is always very close to the isothermal due to the small values of K ($\sim 10^{-3}$). On the contrary, in experiments with water and metal walls the values of K are in the order of magnitude 0.1 (see Kasagi et. al. [7] for detailed values) and the wall thickness becomes a decisive factor. For thick walls of the heater the boundary condition is very close to the isothermal ($K = 0$), and for thin walls the boundary condition is very close to isoflux ($K = \infty$).

3. Heat Transfer Rate at Thermal Activity Ratios *K*=0 and *K*=∞ - Theory

The study of Tiselj et al. [6] discussed the comparison of the isothermal and the isoflux wall boundary conditions for the dimensionless temperature θ^+ in fully developed flume flow at low Reynolds number $Re_\tau = 171$ and at Prandtl numbers $Pr = 1.0$ and $Pr = 5.4$. Large differences in the profiles of the main turbulent statistics (especially temperature RMS fluctuations) were reported by Tiselj et. al. [6] in the near-wall region (conductive sub-layer). However, according to Tiselj et. al. [6], the mean temperature profile did not depend on the type of the thermal boundary condition. Recently, measurements of the water-flume heated first with a thick heater and later with a thin foil of 50 μm were taken by Mosyak et. al. [9]. The experimental setups were very close to the both types of ideal boundary conditions, however they did not reveal any difference in the heat transfer rate.

Another DNS study that considered different types of thermal boundary conditions was performed recently; Kong et al. [10] analyzed the turbulent thermal boundary layer and the influence of thermal boundary condition on the turbulent statistics at $Pr = 0.7$. However, the isothermal boundary condition used by Kong et al. [10] meant constant wall temperature, i.e. isothermal boundary condition for physical temperature T and not for dimensionless temperature θ^+. As a consequence, Kong et al. [10] reported roughly 5 per cent more efficient heat transfer with the isoflux boundary condition for θ^+ than with the isothermal boundary condition for T.

Despite the DNS results of Tiselj et. al. [6], and measurements of Mosyak et. al. [9], it is possible to analytically prove the existence of the different temperature profiles for the isothermal and isoflux boundary conditions for temperature θ^+. For fully developed channel or flume flow the energy equation (1) is averaged in time from $t = 0..\infty$, and in space from $x, z = -\infty..\infty$, and $y = 0..y^+$ as follows (see also Kasagi et al. [1] or Kawamura et. al. [3]):

$$0 = -\left\langle v'^+ \theta'^+ \right\rangle_{t,x,z}(y^+) + \frac{1}{Pr}\frac{d\left\langle \theta^+ \right\rangle_{t,x,z}}{dy^+}(y^+) - 1 + \frac{1}{Re_\tau u_B^+}\int_0^{y^+}\left\langle u^+ \right\rangle_{t,x,z}(y^+)\,dy^+ \,, \tag{3}$$

Eq. (3) shows that the mean temperature profile can be calculated if the mean velocity profile $< u >_{t,x,z}(y^+)$ and wall-normal turbulent heat flux profile $< v'^+ \theta'^+ >_{t,x,z}(y^+)$ are known. It is known[3] that in the wall vicinity, the wall-normal turbulent heat flux for isothermal boundary condition can be expanded in terms of y^+ as:

$$\left\langle v'^+ \theta'^+ \right\rangle_{t,x,z}(y^+)_{ISOTHERMAL} = a_1 y^{+3} + a_2 y^{+4} + \dots\dots\dots . \tag{4}$$

From asymptotical behavior of v'^+ and θ'^+ a similar expansion can be obtained also for the isoflux boundary condition (see Kong et. al. [10]):

$$\left\langle v'^+ \theta'^+ \right\rangle_{t,x,z}(y^+)_{ISOFLUX} = b_1 y^{+2} + b_2 y^{+3} + \dots\dots\dots . \tag{5}$$

Since the near-wall behavior of the wall-normal turbulent heat flux depends on the thermal boundary condition, it is clear from Eq. (3) that the mean temperature profiles also depend on the type of thermal boundary condition. However, the analytical calculation can prove the existence of the difference but cannot give the magnitude of the differences, as the coefficients of the expansion terms (Eq. 4 and 5) cannot be determined analytically.

4. Heat Transfer Rate at Thermal Activity Ratios K=0 and K=∞ - DNS

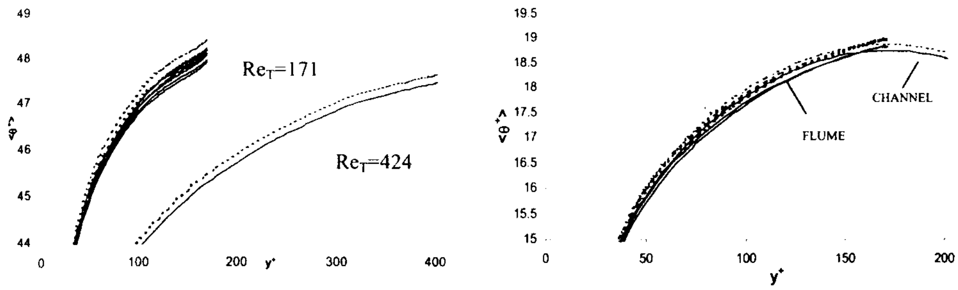
The goal of the present study is to estimate the differences between the temperature profiles calculated with isothermal and isoflux thermal boundary conditions for the dimensionless temperature θ^+, which obviously do exist, but to the best of our knowledge, have not been observed yet [6,9]. Reduction of the DNS statistical uncertainty in the present work allowed the observation of these differences and their magnitudes at friction Reynolds numbers Re_τ = 150, 170, and 424 and Prandtl numbers Pr = 0.025, 0.71, 1.0, and 5.4 (see Table 1). In the previous DNS study of Tiselj et. al. [6] the difference in the mean temperature profiles was attributed to the statistical uncertainty, because the results for each type of boundary conditions were obtained with separate code runs, which produced a slightly different velocity field. The modified computer code used in the present work allows simulations of multiple temperature fields with a single velocity field. This feature of the improved code eliminates the statistical uncertainty of the velocity term in Eq. (3) (last term on the R.H.S), and enables the calculation of the differences in the mean temperature profiles at different thermal boundary conditions, which remained hidden in the previous work of Tiselj et. al. [6].

All DNS tests considered in the present study are summarized in Table 1. They were obtained with simultaneous DNS of a single velocity field and two thermal fields: one with the isothermal and the other with the isoflux boundary condition for the dimensionless temperature θ^+. Four different Prandtl numbers were considered in this study: $Pr = 5.4$ (test cases no. 1-5), $Pr = 1.0$ (test cases 6-9), $Pr = 0.71$ (test cases 10-13), and $Pr = 0.025$ (test case no. 14). Test cases 1-4 and 6-8 were performed for the flume geometry at friction Reynolds number $Re_\tau = 171$, test case 9 for the channel geometry at $Re_\tau = 171$, and test cases 10-14 for the channel geometry at $Re_\tau = 150$.

It is important to stress that the near-wall turbulent heat transfer is not affected by geometry differences between channel and flume flow. The influence of the thermal boundary condition is limited to the conductive sub-layer, while the differences between the channel and flume geometry are limited to the zone near the center of the channel or the top of the flume. This can be seen in Figs. 1-3 comparing the cases 8 and 9 in Table 1. Both cases are performed at the same Reynolds Re_τ and Prandtl number Pr but for a different geometry. The influence of the geometry is seen in Fig. 1b in the mean temperature θ^+ profiles at $y^+ > 70$, while the near-wall behavior is not affected by the geometry as can be seen in Figs. 2 and 3.

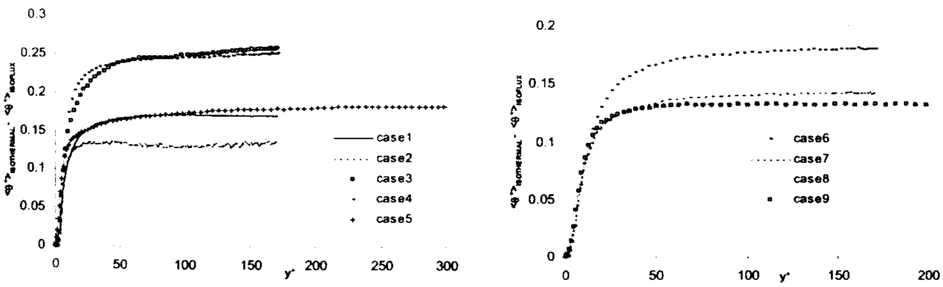
Test case no. 5 was performed at higher friction Reynolds number $Re_\tau = 424$. Kawamura et al.11 reported relatively weak influence of the Reynolds number on the near-wall behavior of the turbulent heat transfer statistics (mean temperature, RMS fluctuations, turbulent heat fluxes) in the turbulent channel. However, their study was limited to the isothermal boundary condition. The results of the test case no. 5 show that conclusions of Kawamura et al.[11] remain valid also for the isoflux boundary condition.

Column no. 10 in Table 1 gives the maximum temperatures in the channel or flume calculated with the isothermal boundary condition for dimensionless temperature θ^+. In the DNS of flume flows the maximum temperatures are obtained at the top of the flume, while in the DNS of channel flows the maximum appears in the middle of the channel. Column no. 11 shows the relative difference of the maximum temperatures calculated with isothermal and isoflux boundary conditions. The heat transfer near the flat wall heated with the same heat flux and cooled with the fluid at the same Reynolds and Prandtl numbers, is between 0.3 and 1.0 per cent more efficient when the isoflux boundary condition is imposed at the wall-fluid interface, than in the case of the isothermal boundary condition for dimensionless temperature θ^+. As these differences are of the same order of magnitude as the statistical uncertainty (0.5%), it is clear, why they were not noticed or measured in the previous studies[6,9]. Figures. 1 present mean temperature θ^+ profiles, whereas Figs. 2 show the differences between the mean temperature profiles that were calculated with the isothermal and isoflux boundary conditions. Figs. 3 show the wall-normal turbulent heat fluxes for each type of boundary conditions. All the differences are accumulated through the conductive sub-layer.



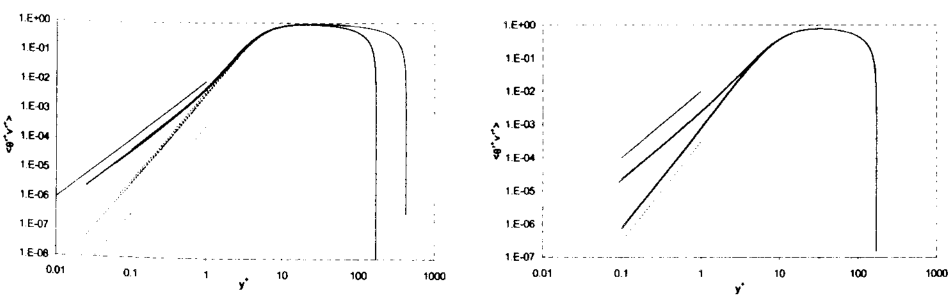
a) $Pr = 5.4$ b) $Pr = 1.0$

Figure 1: Mean temperature profiles at different Prandtl numbers - influence of the thermal boundary condition for dimensionless temperature θ^+. Solid lines: isoflux BC, Dashed lines: isothermal BC. **a)** $Pr = 5.4$ (cases 1,2,3,4,5), **b)** $Pr = 1.0$ (cases 6,7,8,9).



a) $Pr = 5.4$ b) $Pr = 1.0$

Figure 2: Profile of the difference between the mean temperatures calculated with isoflux and isothermal boundary condition for dimensionless temperature θ^+. **a)** $Pr = 5.4$ (cases 1,2,3,4,5), **b)** $Pr = 1.0$ (cases 6,7,8,9).



a) $Pr = 5.4$ b) $Pr = 1.0$

Figure 3: Mean wall-normal heat flux profiles at different Prandtl numbers - influence of the thermal boundary condition for θ^+. Solid lines: isoflux BC, Dashed lines: isothermal BC. **a)** $Pr = 5.4$ (cases 1,2,3,4,5), **b)** $Pr = 1.0$ (cases 6,7,8,9). Straight lines denote theoretical limiting behavior near the wall.

Table 1: Summary of DNS calculations and main results - all quantities in wall units.

1	2	3	4	5	6	7	8	9	10	11
Run No.	Pr	Re_τ	Geo-metry	Computational domain size (x^+,y^+,z^+)	Grid (x,y,z)	Time step	Aver-aging time	Bulk vel. u_B	Max. temp.[a]	$\Delta\theta_{MAX}$ (%)[b]
1	5.4	171	flume	2146x171x537	256x129x128	0.0256	2562	15.41	48.03	0.35
2	5.4	171	flume	2146x171x537	256x129x128	0.0256	2562	15.47	48.18	0.28
3	5.4	171	flume	2146x171x537	128x65x72	0.0256	1537	15.55	48.37	0.54
4	5.4	171	flume	2146x171x537	128x65x72	0.0427	2562	15.52	48.16	0.52
5	5.4	424	flume	2664x424x1332	256x129x192	0.0318	6360	17.54	46.70	0.33
6	1	171	flume	2146x171x537	128x65x72	0.0512	5124	15.47	19.00	0.95
7	1	171	flume	2146x171x537	128x65x64	0.0512	5124	15.45	18.99	0.75
8	1	171	flume	2146x171x537	128x65x64	0.0512	5124	15.43	18.84	0.70
9	1	171	chan.	2146x342x537	128x97x64	0.0854	6832	15.48	18.85	0.67
10	0.71	150	chan.	2356x300x942	128x97x128	0.09	3600	15.24	15.66	0.75
11	0.71	150	chan.	4712x300x471	256x97x64	0.09	9000	15.29	15.70	0.65
12	0.71	150	chan.	2356x300x471	128x97x64	0.12	7200	15.21	15.63	0.66
13	0.71	150	chan.	2356x300x942	192x129x160	0.06	4800	15.25	15.59	0.71
14	0.025	150	chan.	2356x300x942	128x97x128	0.09	4500	15.25	1.930	0.55

[a] Max. mean temperature for isotherm. BC $<\theta^+>_{MAX\_ISOTHERMAL} = <\theta^+>_{t,x,z}\ (y^+ = \delta^+)$ (appears in the centerline of the channel and on the free surface of the flume).

[b] $\Delta\theta_{MAX} = 100 * (1 - <\theta^+>_{MAX\_ISOFLUX} / <\theta^+>_{MAX\_ISOTHERMAL})$ %

5. Conclusions

It is possible to analytically show the existence of the difference in the heat transfer rates between the isoflux and isothermal boundary conditions for dimensionless temperature, which are applicable for $K = \infty$ and $K = 0$, respectively. However, the magnitude of the difference cannot be calculated analytically. The present DNS results show roughly 0.5% higher heat transfer rate for the isoflux boundary condition in the range of low friction Reynolds numbers $Re_\tau = 150$, 171 and 424, at Prandtl numbers $Pr = 0.025$, 0.71, 1.0 and 5.4. Although, the results of this study are limited to the low Reynolds numbers $Re_\tau = 150$-424 and Prandtl numbers between 0.025 and 5.4, we assume that the same small differences in heat transfer appear also at other Reynolds and Prandtl numbers.

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