Mathematical Description of Multiphase Flow During Meltdown Scenario

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Abstract

Melting ultimately requires the heat production rate to exceed the removal rate. In a nuclear reactor this situation is initiated by either overpower or undercooling conditions. The latter is related to reduced cooling through loss of coolant flow or loss of coolant itself and may lead to general meltdown of the reactor core.

Molten material is accumulated in the lower plenum of the reactor pressure vessel. The separation of phases results from different components densities of molten material. High temperature molten UO_2 will slowly melt through the wall of RPV lower plenum. To avoid disruption of RPV, an instantaneous heat transfer is needed.

To simulate the situation described above, general multiphase flow equations were developed. In order to describe the multiphase flow fully, the continuity, momentum and energy equations were derived using ensemble averaging rather then time or spatial averaging. Because of geometry of the lower plenum a spherical coordinate system was used, to enhance the accuracy of the calculation on the border.

Introduction

In the case of prolonged and complete failure of normal and emergency coolant system of nuclear reactor, the fission material decay heat causes overheat conditions. The rise in temperature brings steel material of the reactor core and zircaloy cladding to exothermic oxidation at 1300 K [1]. If cooling is not restored, the general meltdown of the reactor core takes place. Melting scoops the steel equipment at 1700 K [2], then the zirconium oxide at 2990 K [3] and finally the uranium oxide at 3113 K [4]. Such conditions produce jets of melt, which accumulate in the lower plenum of the reactor core.

Because of different melting points of materials in the reactor core, the meltdown process may be divided in two stages: melting of steel/zircaloy components and melting of uranium and zirconium oxides. The large amount of steel component in reactor core (29500kg [5]) and its low melting point make the contribution of other components in the first stage of melting negligible. The same can be stated for molten uranium oxide in the second stage of melting. This are the reasons why such complex problem may be simplified to the mixing of two phases. The third phase represents the RPV wall that contacts high temperature uranium oxide melt, which results in gradual melting and erosion.

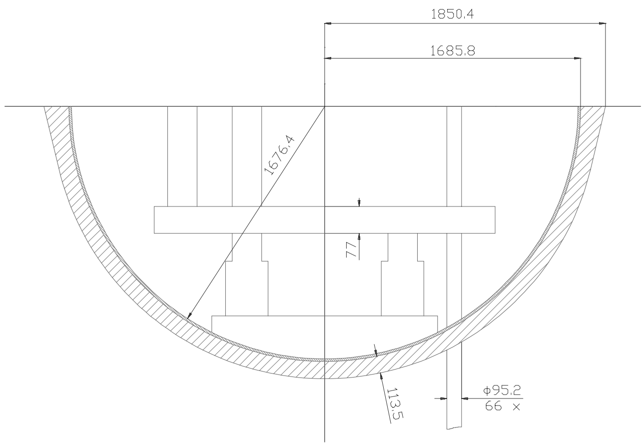

Figure 1: Lower plenum of RPV

Our intention is to evaluate cooling required to avoid disruption of RPV and release of core material. Secondly we intend to proof usefulness and reliability of the probabilistic calculation method which was developed for steam explosion multiphase flow simulation [6].

Mathematical Description

Every multiphase flow can be described with basic hydrodynamic equations for single phase flow. These are

- continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla (\rho \vec{v}) = 0 \quad , \tag{1}$$

- momentum equation

$$\frac{\partial (\rho \vec{v})}{\partial t} + \nabla (\rho \vec{v} \otimes \vec{v}) = - \nabla p + \Delta \underline{\tau} + \rho \vec{g} \quad , \tag{2}$$

- energy equation

$$\frac{\partial (\rho h)}{\partial t} + \nabla \cdot (\rho h \vec{v}) = \frac{\partial p}{\partial t} + \vec{v} \cdot \nabla p + \underline{\tau} : (\nabla \otimes \vec{v}) + \nabla \cdot (\lambda \nabla T) + q \quad , \tag{3}$$

- equations of state

$$\rho = \rho\left(\vec{r}, t, T\right) \quad , \tag{4}$$
$$\nu = \nu\left(\vec{r}, t, T\right) \quad ,$$
$$c_p = c_p\left(\vec{r}, t, T\right) \quad ,$$
$$\lambda = \lambda\left(\vec{r}, t, T\right) \quad ,$$
$$q = q\left(\vec{r}, t\right) \quad .$$

In order to treat a multiphase flow, the above equations (1,2,3) have to be written for every single phase of multiphase flow separately, together with interfacial equations on the phase boarders.

Such a system of equations with appropriate interfacial conditions is unsolvable even for single phase flow with a high value of **Re** number. Solving them for multiphase flow where the separate set of equations (1,2,3,4) together with their boundary conditions is valid only in a region, which is confined to moving phase borders in a specific period of time, is only harder. Therefore the equations (1,2,3,4) with instantaneous local values have to be replaced with averaged equations. Most commonly time or spatial averaging are used, which have some disadvantages because the accuracy is very much dependent on the size of reference time or spatial interval respectively. To avoid this weakness and enhance accuracy we used the statistical averaging.

Using statistical averaging, the physical quantities from equations (1,2,3,4) are averaged over an ensemble of imaginary samples. The indication of a separate phase is marked with a characteristic function $\boldsymbol{X_k}$. If the sample is taken from phase $\boldsymbol{k}$, the characteristic function $\boldsymbol{X_k}$ obtains logical value **1** or logical value **0** in opposite situations:

$$X_k\left(\vec{r}, t\right) = \begin{cases} \text{phase } k \text{ sample} \\ \text{not phase } k \text{ sample} \end{cases} \quad . \tag{5}$$

This imaginary sampling is repeated. In the limit, when the number of imaginary samples tends to infinity, the probability of presence for a separate phase $\boldsymbol{k}$ is obtained.

$$\alpha_k\left(\vec{r}, t\right) \cdot \overline{f}\left(\vec{r}, t\right) = \overline{X_k \cdot f\left(\vec{r}, t\right)} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} X_{k,i}\left(\vec{r}, t\right) \cdot f\left(\vec{r}, t\right) \quad . \tag{6}$$

In order to get probabilistic equations (7,8,9) the equations (1,2,3) have to be multiplied with the characteristic function $\boldsymbol{X_k}$ and then averaged over the ensemble. Instead of instantaneous local quantities the averaging quantities are set in the equations together with presence phase probability $\boldsymbol{\alpha_k}$ for a separate phase:

- continuity equation

$$\frac{\partial\left(\alpha_k \rho_k\right)}{\partial t} + \nabla \cdot \left(\alpha_k \rho_k \vec{v}_k\right) = \Gamma_k + C_k \quad . \tag{7}$$

- momentum equation

$$\alpha_k \rho_k \frac{\partial \vec{v}_k}{\partial t} + \alpha_k \rho_k \left(\vec{v}_k \cdot \nabla\right) \vec{v}_k = -\alpha_k \nabla p_k + \alpha_k \rho_k \vec{g} + \alpha_k \vec{M} + \Delta\vec{v}_k \Gamma_k \quad . \tag{8}$$

- energy equation

$$\alpha_k \rho_k \frac{\partial h_k}{\partial t} + \alpha_k \rho_k \left(\vec{v}_k \cdot \nabla\right) h_k = \alpha_k E_k + \Delta h_k \Gamma_k \quad . \tag{9}$$

The disadvantage of described procedure is the formation of cofluctuation tensors where deviations from averaged physical quantities are collected. In this case they are denoted under the terms $\boldsymbol{C_k}$, $\boldsymbol{M_k}$ and $\boldsymbol{E_k}$, whereas $\boldsymbol{\Gamma_k}$ describes the interfacial mass transport. The real art of presented method is determination of cofluctuation tensors. Because it is impossible to calculate them analytically, they have to be evaluated with a suitable probability distribution [8].

To complete set of equations, one considers the fact that sum of all presence phase probabilities is always **1**

$$\sum_{k=1}^{n} \alpha_k \left(\vec{r}, t\right) = 1 \quad , \tag{10}$$

and the pressure is equal in all phases

$$p_1 = p_1 = ... = p_n = p \quad . \tag{11}$$

The system of equations in now completed. Boundary and initial conditions will be shown later in the text.

Methods of Solution

The geometry of RPV lower plenum required the use of spherical coordinates. Because of the axysymmetry we were able to reduce the mathematical description to two dimensions. Fully developed equations in spherical coordinate system have now acquired a much more exacting form:

- continuity equation

$$\rho_k \frac{\partial \alpha_k}{\partial t} + \alpha_k \frac{\partial \rho_k}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \cdot \alpha_k \rho_k v_{k,r}\right) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \cdot \alpha_k \rho_k v_{k,\theta}\right) = \Gamma_k + C_k \quad , \tag{12}$$

- momentum equation in $\boldsymbol{r}$ direction

$$\alpha_k \rho_k \frac{\partial v_{k,r}}{\partial t} + \alpha_k \rho_k \left(v_{k,r} \frac{\partial v_{k,r}}{\partial r} + \frac{v_{k,\theta}}{r} \frac{\partial v_{k,r}}{\partial \theta} - \frac{v_{k,\theta}^2}{r} \right) = = - \alpha_k \frac{\partial p}{\partial r} - \alpha_k \rho_k g \cdot \cos\theta + \Gamma_k \Delta v_{i,r} + \alpha_k M_{k,r} \quad , \tag{13}$$

- momentum equation in $\boldsymbol{\theta}$ direction

$$\alpha_k \rho_k \frac{\partial v_{k,\theta}}{\partial t} + \alpha_k \rho_k \left(v_{k,r} \frac{\partial v_{k,\theta}}{\partial r} + \frac{v_{k,\theta}}{r} \frac{\partial v_{k,\theta}}{\partial \theta} - \frac{v_{k,r} v_{k,\theta}}{r} \right) = = - \frac{\alpha_k}{r} \frac{\partial p}{\partial \theta} + \alpha_k \rho_k g \cdot \sin\theta + \Gamma_k \Delta v_{i,\theta} + \alpha_k M_{k,\theta} \quad , \tag{14}$$

- energy equation

$$\alpha_k \rho_k \frac{\partial h_k}{\partial t} + \alpha_k \rho_k \cdot \left(v_{k,r} \frac{\partial h_k}{\partial r} + \frac{v_{k,\theta}}{r} \frac{\partial h_k}{\partial \theta} \right) = \alpha_k E_k + \Delta h_i \Gamma_k \qquad . \tag{15}$$

To perform calculation of all physical quantities in the equations (12,13,14,15) one needs to developed corresponding algorithm. For this purpose the continuity equation has to be transformed to a different form. When continuity equation is rewritten for all present phases together as a sum of the particular continuity equations (7) and equation (10) is considered, the time derivative of phase probabilities is :

$$\sum_{k=1}^{n} \frac{\partial \alpha_k}{\partial t} = \frac{\partial}{\partial t} \sum_{k=1}^{n} \alpha_k = 0 \qquad . \tag{16}$$

Moreover, the continuity equations can be simplified if we estimate the size of total density derivative realistically.

$$\frac{D \rho_k}{D t} = 0 \qquad . \tag{17}$$

The derived form of the continuity equation:

$$\sum_{k=1}^{n} \nabla \left(\alpha_k \vec{v}_k \right) = 0 \qquad , \tag{18}$$

is now coupled to momentum equation (7) and fully developed. This equation (19) conform pressure field to an already calculated velocity field so that both satisfy continuity equation.

$$-\sum_{k=1}^{n} \frac{\alpha_k}{\rho_k}\cdot\left(\left(\frac{2}{r}\frac{\partial p}{\partial r}+\frac{\partial^2 p}{\partial r^2}+\frac{1}{r^2\tan\theta}\frac{\partial p}{\partial\theta}+\frac{1}{r^2}\frac{\partial^2 p}{\partial\theta^2}\right)-\frac{\partial p}{\partial r}\cdot\left(\alpha_k\frac{\partial}{\partial r}\left(\frac{1}{\rho_k}\right)+\frac{1}{\rho_k}\frac{\partial\alpha_k}{\partial r}\right)\right) \tag{19}$$

$$-\sum_{k=1}^{n}\frac{\partial p}{\partial\theta}\cdot\left(\frac{\alpha_k}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\rho_k}\right)+\frac{1}{\rho_k r^2}\frac{\partial\alpha_k}{\partial\theta}\right)=$$

$$=\sum_{k=1}^{n}\left(\frac{2}{r}\left(\alpha_k v_{k,r}\right)+\frac{\partial\left(\alpha_k v_{k,r}\right)}{\partial r}+\frac{1}{r\cdot\tan\theta}\left(\alpha_k v_{k,\theta}\right)+\frac{1}{r}\frac{\partial\left(\alpha_k v_{k,\theta}\right)}{\partial\theta}\right)\cdot\left(\frac{2}{r}v_{k,r}+\frac{\partial v_{k,r}}{\partial r}+\frac{1}{r\cdot\tan\theta}v_{k,\theta}+\frac{1}{r}\frac{\partial v_{k,\theta}}{\partial\theta}\right)$$

$$+\sum_{k=1}^{n}v_{k,r}\cdot\left(-\frac{2}{r^2}\left(\alpha_k v_{k,r}\right)+\frac{2}{r}\frac{\partial\left(\alpha_k v_{k,r}\right)}{\partial r}+\frac{\partial^2\left(\alpha_k v_{k,r}\right)}{\partial r^2}-\frac{1}{r^2\tan\theta}\left(\alpha_k v_{k,\theta}\right)\right)$$

$$+\sum_{k=1}^{n}v_{k,r}\cdot\left(\frac{1}{r\cdot\tan\theta}\frac{\partial\left(\alpha_k v_{k,\theta}\right)}{\partial r}-\frac{1}{r^2}\frac{\partial\left(\alpha_k v_{k,\theta}\right)}{\partial\theta}+\frac{1}{r}\frac{\partial^2\left(\alpha_k v_{k,\theta}\right)}{\partial\theta\,\partial r}\right)$$

$$+\sum_{k=1}^{n}v_{k,\theta}\cdot\left(\frac{2}{r^2}\frac{\partial\left(\alpha_k v_{k,r}\right)}{\partial\theta}+\frac{1}{r}\frac{\partial^2\left(\alpha_k v_{k,r}\right)}{\partial r\,\partial\theta}-\frac{\alpha_k v_{k,\theta}}{r^2\sin^2\theta}+\frac{1}{r^2\tan\theta}\frac{\partial\left(\alpha_k v_{k,\theta}\right)}{\partial\theta}+\frac{1}{r^2}\frac{\partial^2\left(\alpha_k v_{k,\theta}\right)}{\partial\theta^2}\right)$$

$$+\sum_{k=1}^{n}\frac{\alpha_k}{r^2}\cdot\left(2\,r\,v_{k,r}\frac{\partial v_{k,r}}{\partial r}+\left(r\frac{\partial v_{k,r}}{\partial r}\right)^2+r^2\,v_{k,r}\frac{\partial^2 v_{k,r}}{\partial r^2}+v_{k,\theta}\frac{\partial v_{k,r}}{\partial\theta}+r\frac{\partial v_{k,\theta}}{\partial r}\frac{\partial v_{k,r}}{\partial\theta}+r\,v_{k,\theta}\frac{\partial^2 v_{k,r}}{\partial\theta\,\partial r}-v_\theta^2-2\,r\,v_\theta\frac{\partial v_{k,\theta}}{\partial r}\right)$$

$$+\sum_{k=1}^{n}\frac{\alpha_k}{r\cdot\sin\theta}\cdot\left(\cos\theta\;v_{k,r}\frac{\partial v_{k,\theta}}{\partial r}+\sin\theta\;\frac{\partial v_{k,r}}{\partial\theta}\frac{\partial v_{k,\theta}}{\partial r}+\sin\theta\;v_{k,r}\frac{\partial^2 v_{k,\theta}}{\partial r\,\partial\theta}\right)$$

$$+\sum_{k=1}^{n}\frac{\alpha_k}{r\;\sin\theta}\cdot\left(\cos\theta\frac{v_{k,\theta}}{r}\frac{\partial v_{k,\theta}}{\partial\theta}+\frac{\sin\theta}{r}\left(\frac{\partial v_{k,\theta}}{\partial\theta}\right)^2+\sin\theta\frac{v_{k,\theta}}{r}\frac{\partial^2 v_{k,\theta}}{\partial\theta^2}\right)$$

$$+\sum_{k=1}^{n}\frac{\alpha_k}{r\cdot\sin\theta}\cdot\left(\cos\theta\frac{v_{k,r}\,v_{k,\theta}}{r}+\sin\theta\frac{v_{k,\theta}}{r}\frac{\partial v_{k,r}}{\partial\theta}+\sin\theta\frac{v_{k,r}}{r}\frac{\partial v_{k,\theta}}{\partial\theta}\right)$$

$$+\sum_{k=1}^{n}\frac{\partial\alpha_k}{\partial r}\cdot\left(\left(v_{k,r}\frac{\partial v_{k,r}}{\partial r}+\frac{v_{k,\theta}}{r}\frac{\partial v_{k,r}}{\partial\theta}-\frac{v_{k,\theta}^2}{r}\right)+\frac{1}{r}\frac{\partial\alpha_k}{\partial\theta}\cdot\left(v_{k,r}\frac{\partial v_{k,\theta}}{\partial r}+\frac{v_{k,\theta}}{r}\frac{\partial v_{k,\theta}}{\partial\theta}+\frac{v_{k,r}\,v_{k,\theta}}{r}\right)\right)$$

$$-\sum_{k=1}^{n}\alpha_k\cdot\left(\frac{\partial}{\partial r}\left(\frac{1}{\rho_k}\right)M_{k,r}+\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{1}{\rho_k}\right)M_{k,\theta}\right)$$

$$-\sum_{k=1}^{n}\frac{\alpha_k}{\rho_k}\cdot\left(\left(\frac{2}{r}M_{k,r}+\frac{\partial M_{k,r}}{\partial r}+\frac{1}{r\cdot\tan\theta}M_{k,\theta}+\frac{1}{r}\frac{\partial M_{k,\theta}}{\partial\theta}\right)-\frac{1}{\rho_k}\cdot\left(\frac{\partial\alpha_k}{\partial r}M_{k,r}+\frac{1}{r}\frac{\partial\alpha}{\partial\theta}M_{k,\theta}\right)\right)$$

Initial and boundary conditions

Initial and boundary conditions complete the above hydrodynamic equations (12,13, 14, 15 and 19) into a fully defined mathematical problem. Geometry that was chosen for numerical simulation is a simplification of realistic arrangement of RPV lower plenum equipment.

In a round basin of RPV lower plenum there are already stratified layers of 5 m^3 of molten fuel UO2 and 5 m^3 of molten steel. Because of much greater density (10960 kg/m^3 [4]) of the fuel, it layers at the bottom and molten steel (5180 kg/m^3 [2]) at the top.

Initial temperature of molten steel is 2000 K and the temperature of molten fuel is 3200 K. Because of axysymmetry, the problem is simplified to two dimensions. The numerical mesh has to cover only a half of the containment cross section.

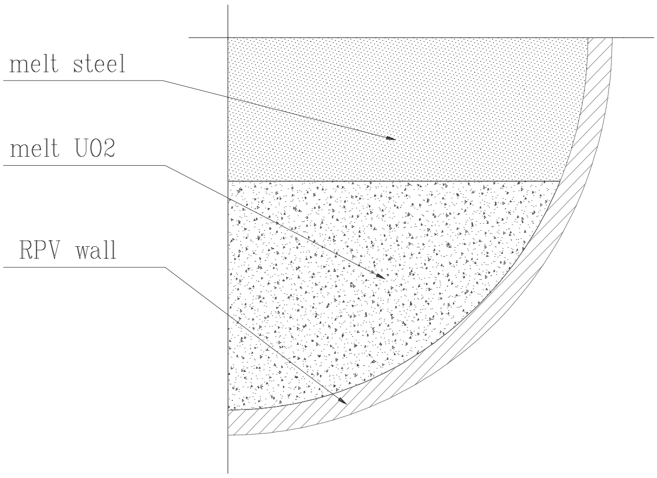

Figure 2: Initial conditions of multiphase mixture

To solve successfully the equations (12,13,14,15 and 19), the boundary conditions are also important. The figure 3 shows how they are defined around whole calculating area.

Conclusions

This paper presents a mathematical model with derivation of multiphase probabilistic equations together with initial and boundary conditions for a specific problem. The method was used before successfully to simulate behavior of a multiphase mixture during the premixing phase of steam explosion [9]. The mathematical problem was solved using Alternating Direction Implicit scheme in the case of elliptic pressure equation (19) and Lax-Wendroff scheme in case of time depended continuity (12), momentum (13,14) and energy (15) equation.

In the presented problem, we seek the numerical solution in same direction, although the strong convective movement of multiphase mixture is dominant. Successful simulation of the cooling problem would confirm the developed probabilistic method for multiphase treatment and numerical tools for its successful solving.

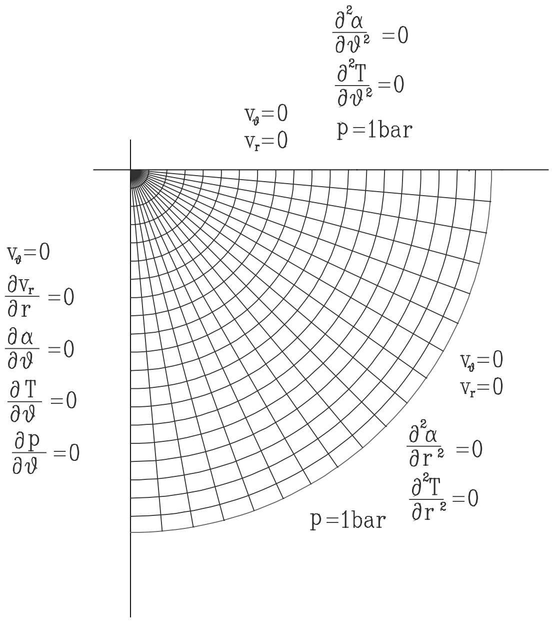

Figure 3: Boundary conditions of numerical mesh

Nomenclature

Latin

c_p	heat capacity
C	cofluctuation tensor
E	cofluctuation tensor
g	gravity
h	enthalpy
M	cofluctuation tensor
p	pressure
q	heat generation
r	radial coordinate
t	time
T	temperature
v	velocity
X	characteristic function

Greek

α	presence probability
Γ	mass generation
θ	cylindrical coordinate
λ	thermal conductivity
ν	kinematic viscosity
τ	shear tensor

Subscript

k	phase index
i	interface index
r	vector component
θ	vector component

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