

# A Fully Coupled Numerical Analysis of the Response of a Circular Membrane to High Wind Loads

#### Andrej Horvat & Adam Kyte

Intelligent Fluid Solutions Ltd. 127 Crookston Road, London, SE9 1YF Tel./Fax: +44 (0)1235 819 729 E-mail: andrej.horvat@intelligentfluidsolutions.co.uk

#### Abstract

A fluid-structure analysis of a test case representing a radio antenna that is subjected to wind load is performed. Due to high wind speeds, large deflection of the protective fabric occurs, which may damage the sensitive electronic equipment. Therefore, accurate prediction of maximum fabric deflection under the most severe wind conditions is of vital importance. The analysis is performed for the wind speed of 155 mph.

The maximum deflection of the protective fabric is calculated using four alternative methods, including a 2-way coupled fluid-structure simulation. Issues that the authors encountered performing the numerical simulations are also discussed. The comparison of the results shows that, for the studied case, a difference of approximately 12% exists between the full 2-way coupled solution of the fluid-structure system and a simple estimate of the maximum deflection using algebraic expressions. The level of deviation increases with increasing fabric deflection (e.g. lower elasticity modulus, higher wind speed) and non-symmetrical pressure load (e.g. change of the antenna design).

## Introduction

In certain flow situations, the action of the fluid flow leads to a deformation of the solid structure that is in contact with. If the deformation process is slow in comparison to the flow, both systems can be treated separately, as the flow is only marginally influenced by the deforming structure. On the other hand, if the rate of deformation is of the same order or even faster than the fluid flow response, both systems have to be treated as a coupled system. This significantly complicates the analysis of such a system.

The present paper discusses a particular fluid-structure system and compares the results from alternative analysis approaches. The subject of analysis is a radio-antenna, which is exposed to wind of 155 mph. The sensitive electronics of the antenna is protected with a deformable nylon fabric. Our task was to calculate wind loading and associated fabric deflection for a range of alternative antenna designs. This paper presents the results from a test case (Fig. 1) that was developed to evaluate the most appropriate methodology to use. The antenna designs as well as their engineering details have been omitted to protect commercial confidentiality.



#### Figure 1: Problem definition

In the test case, the antenna is replaced with a cylinder with diameter D = 2.46 m and height H = 2.0 m. The protective fabric has a thickness  $\delta = 0.45$  mm. On the rim of the cylinder, the fabric is simply supported, with deformations restricted in all three directions although bending is permitted. The elasticity (Young's) modulus of the membrane is E = 100.0 N/mm<sup>2</sup>.

To evaluate such a coupled fluid-structure system (Fig. 1), one needs to solve conservation equations for fluid mass and momentum:

$$\partial_i v_i = 0 \tag{1}$$
$$\rho v_j \partial_j v_i = -\partial_i p + \mu \partial_j \partial_j v_i$$

and equations of force balance, specific deformation and Hooke's law [1]:

$$\partial_{k}\sigma_{ik} + f_{i} = 0$$

$$\varepsilon_{ik} = \frac{1}{2} (\partial_{k}u_{i} + \partial_{i}u_{k} + \partial_{k}u_{l}\partial_{i}u_{l})$$

$$\sigma_{ik} = \frac{E}{1 + \nu} \left( \varepsilon_{ik} + \frac{\nu}{1 - 2\nu} \varepsilon_{ll}\delta_{ik} \right)$$
(2)

where  $v_i$  is velocity, p is pressure,  $u_i$  is displacement,  $\varepsilon_{ik}$  is strain, and  $\sigma_{ik}$  is stress. The remaining terms are material or mechanical properties. These two sets of equations are coupled through pressure p and displacement  $u_i$  on the interface. As we are only interested in a final and steady deflection of the fabric, Eqns. (1) and (2) are simplified to their steady state form. Also at these flow speeds, the fluid can be considered to be incompressible (1). In

general, the turbulence equations also need to be solved. However, in the interests of clarity, turbulence modelling is omitted in the paper.

The fluid flow equations (1) and structural mechanics equations (2) are non-linear. In some cases, they can be simplified:

- for small deformations, the strain relations (2) can be linearised,
- for large pre-stress imposed onto fabric, Eqns. (2) can be simplified to a single Laplace equation, which can be solved analytically

However, neither of these assumptions is valid in our case, and their use would produce a large error. So, what are the tools available to solve a coupled fluid-structure problem, where the deformation of structure is large?

#### Methodology and Associated Issues

The above described fluid-structure interaction problem can be evaluated using an approximate analytical model. The pressure load caused by the wind, which is directed perpendicular to the fabric surface, can be approximated using Bernoulli 's equation:

$$\Delta p = \frac{1}{2} \rho u_{inlet}^2 \tag{3}$$

Although, structure mechanics equations (2) do no have an analytical solution for the discussed test case, a close form approximation for maximum deflection can be found in the literature [2]:

$$u_{y,max} = 0.2627 \left(\frac{\Delta p D^4}{E\delta}\right)^{\frac{1}{3}}$$
<sup>(4)</sup>

where  $\delta$  is fabric thickness.

More complete and precise results can be obtained only with numerical simulation. When building a numerical model, one can use different levels of complexity. For the present case, we tested the following numerical methods:

- pressure load approximation (2) & deflection calculation with ANSYS Simulation
- pressure field calculation with ANSYS CFX & deflection calculation with ANSYS Simulation (1-way coupled)
- pressure field calculation with ANSYS CFX & deflection calculation with ANSYS Simulation (2-way coupled)

The numerical mesh used in the numerical simulations is shown in Fig. 2. The mesh contained approximately 76,000 nodes and was used for these test cases only.



Figure 2: Numerical mesh

The impression that a user can simply include all the elements of fluid and structural mechanics in a simulation model and then press a button to obtain results is not correct and is significantly over-simplified. Due to the low fabric stiffness, all 1-way and 2-way coupled steady-state simulations of our test case failed. Transient simulations did work, but required the use of a very small time-step. Typically, the results from the transient analyses showed a slowly decaying oscillatory motion (Fig. 3). We were able to identify 3 different modes of the fabric vibration Qualitatively, they correspond to the first eigenvalues of the Bessel function  $J_0$  and  $J_1$  [3].



Figure 3: Time variation of total deflection for one of the 1-way coupled cases

Unfortunately, due to the long decay period and required short time-steps, the calculation of the converged steady-state for such system would require prohibitively large computational resources. Therefore, to improve the convergence times, the mechanical properties of the fabric were altered in a way that does not influence the final deflection, but it does improve the transient behaviour. For this purpose, the density of the fabric was increased and damping was introduced. This setup gave a robust convergence of the 1-way and the 2–way coupled systems. A comparison of the results from the analyses is presented in the next section.

### **Results and discussion**

All the reported numerical simulations of air flow were performed with ANSYS CFX simulation package, whereas the structural response of the fabric was calculated with ANSYS Simulation software. Both packages are part of ANSYS Workbench environment.

Initial transient simulations of air flow around the antenna showed that a steady-state flow field does exist for the investigated wind speed. This result gave us the confidence to alter the mechanical properties of the protective fabric in order to accelerate the solution on the structural side without compromising the final result. Figure 4 shows the steady-state pressure field of the 2-way coupled FSI simulation and Fig. 5 the associated velocity field.



Figure 4: Pressure field from the 2-way coupled FSI simulation



**Figure 5:** Velocity vector field from the 2-way coupled FSI simulation

One can observe a large flow stagnation area with a high pressure region that is formed near the antenna's protective fabric. Behind the antenna, the pressure field is depressed and a large toroidal recirculating flow structure is formed. Figure 5 also shows the deformed fabric with displacement as a contour plot.

To estimate the accuracy of the alternative analyses methodologies, the maximum deflection predicted by the analyses were compared. Table 1 summarises the deflection values and their deviation from the 2-way coupled FSI solution.

	Max deflection [mm] at 155 mph	Relative discrepancy [%] from 2-way coupled FSI
Pressure load approx. & deflection approx.	317.7	-11.6
Pressure load approx. & deflection (ANSYS Simulation)	316.1	-12.0
Pressure field (CFX) & ANSYS simulation (1-way coupled)	356.4	-0.83
Pressure field (CFX) & ANSYS simulation (2-way coupled)	359.4	-

**Table 1:** Comparison of calculated deflection using different methods

The first row of Table 1 shows the values for an average pressure load approximation via Bernoulli's equation (3) and maximum deflection estimate (4). In the second row, the average pressure load (2) is used in the ANSYS Simulation package. The third and the fourth row present the results of the 1-way coupled and 2-way coupled simulations, respectively.

The calculated deflections span the range from 316 mm to 359 mm. The comparison shows that for this particular test case, the 1-way couple and 2-couple simulations produce almost identical results. Large deviation (approx. 12%) occurs when the distributed pressure load is substituted with a uniform average value. Therefore, the differences between these methods become larger when the load is not uniformly distributed or if the response of the structure is non-symmetric.

## Conclusions

An analysis of structural response was performed for a protective fabric of a radio antenna, which is subject to winds of 155 mph. Four alternative analysis methodologies were used to calculate the applied pressure load and resulting maximum deflection of the fabric. The simplest method employs the algebraic expressions to estimate maximum deflection, whereas the most complex method uses two parts of the ANSYS Workbench package to obtain a fully coupled solution.

Due to relatively large loads and low stiffness of the fabric, the 1-way and 2-way coupled simulations were found to be very demanding. Although, the transient simulation was possible, it was not practical due to the presence of slowly decaying high frequency oscillations. The steady-state solution was found within a reasonable computational time by altering the mass of the fabric and by introducing artificial damping.

The comparison of the calculated maximum deflections showed that for the given test case, the discrepancy between the algebraic approximation of the maximum deflection and the 2-way coupled solution was approximately 12%. The discrepancy between both methods increases for larger fabric deflection or if the load is non-symmetric. In such cases the 2-way coupled fluid-structure analysis is required to capture the complex flow and mechanical behaviour.

# Nomenclature

D	diameter
Ε	elasticity (Young's) modulus
Н	height
р	pressure
$u_i$	displacement
Vi	velocity
δ	thickness
$\varepsilon_{ik}$	strain
μ	dynamic viscosity
ν	Poisson's ratio
ρ	density
$\sigma_{ik}$	stress

## References

- 1. L.D. Landau, E.M. Lifshitz, Theory of Elasticity, 3rd Ed., Butterworth-Heinemann, 2007, p. 53.
- 2. E.J. Hearn, Mechanics of Materials, 3rd Ed., Butterworth-Heinemann, 199
- 3. I.N., Bronshtein, K.A., vSemendyayev, Handbook of Mathematics, 3rd Ed., Springer, p.410.